FLUTTER OF A FINITE RECTANGULAR WING'D MILL IN PITCH AND HEAVE

S.P. Farthing, Wing'd Pump, www.econologica.org, N. Saanich B.C. V8L 5P2 Canada spfd@cantab.net
Accepted by Journal of Power and Energy IMechE July 29 2021.

Exact solutions of flutter are explored for a mode suitable for an oscillating 2D hydrofoil version of the summarised FlutterWing windpump which radically surpasses the old rotary fanpump. The imaginary part ***G*** of the 1D Theodorsen wake lift factor ***T*** rises singularly from the steady to allow 1D pure pitch flutter around the leading edge at low frequency. But such logarithmly infinite negative pitch damping by the shed vorticity is capped by the logarithm of the 2D span. The downwash of an oscillating trailing (tip) vortex is reduced by a function of the reduced frequency based on spanwise distance. Its lag damps pitch and further raises the pitch unstable aspect ratio A to above 36. All binary pitch & heave neutral flutter frequency contours in (inertia, imbalance) space still pass through a nexus of the same total pitch inertia and imbalance as the virtual mass concentrated at ¾ chord, a very high total in water. The internal bound windwise vortex from ¼ chord to trailing edge reduces as A^{-2} the 1D lift slope at all frequencies. It expands the double root of high frequency contours kissing the low frequency limit line at the nexus to make them dip below the lfl line to the right of the nexus as hyperbolae asymptoting to the now lower ray of infinite frequency. Thus low A discovers a new wedge of high frequency binary flutter below the lfl line with real mass moments less than the virtual, as in water.

INTRODUCTION

As early aircraft increased in speed, novel self-amplifying coupled oscillations between control surfaces and wings or rear fuselages caused many structural failures and crashes. For instance the inertia of an unbalanced aileron acting at a cg behind its hinge makes it overshoot the wing at the top of a bending oscillation, so dropping the wing lift to amplify its rebound down. By 1960 flutter instability could be eliminated by dynamic massbalancing, stiffening controls and airframes, and extensive linear computation of the myriad of flutter modes exceeding all other aircraft design calculation.

Duncan [1] had pedantically demonstrated that the airstream powers flutter's unstable oscillations with his model "wing engine", a wing balanced but articulated to pitch in set quadrature to heave (plunge). The BBC 1976 Young Scientists' more practical pitch and roll (Fig 1) free flutter model promised better wind waterpumping than fanmill windpumps without the dangerous climbs. but again had no pump or storm protection. (Pocklington School attributed the idea to Sir. G.I. Taylor, but his biographer G.K Batchelor FRS could not find it in GIT's papers). A sprung model with a cam track to depress a honed piston pumped to high psi. in Southampton U windtunnel. The power varies with amplitude squared in linear theory, but pitch saturates at ±90° and ultimately the power must vary as the swept area or large roll amplitude. In 1993 full-scale smoke [6] found a beneficial extraordinary leading edge vortex shed [7] at full ±90° pitch flip. Definitive confirmation of economic power far outweighing the fatigue penalties of 40% with steel and 20% with wood came with this fullscale prototype base #7 on a test well with instruments read by a computer in a trailer at the BC Science Council [2]. Altogether 8 bases and 6 wings were prototyped. Scaling shows all inertially unbalanced fluid machines (incl.H Vawts and flapping orthinopters) have an upper limit to design flowspeed as the square root of the strength to weight ie the endurance limit stress/ material density[2] and so the FWP niche is a light wind regime (and also gravity-dominated large size)

Sir James Lightill FRS stressed the need for storm protection In 1980 storm stability of- pitch and roll flutter had been hypothesized and ~~was~~ proven successively with models on top of a car and then in Southampton, CEGB Marchwood and UCL windtunnels and,by computations at Gifford & Partners and later in full size prototype #3 in gales and finally algebraically [2]. Very crudely the pitch response to the inclination of the tailheavy wing is decreased by the rise in pitch damping as windspeed so the pitch amplitude decreases with windpseed. avoiding high wing loadings, the windmill low drag equivalent of the high thrust efficiency of oscillating propulsion in nature. Gust response goes negative at cutout and is more and more damped above. The cutout wing feathers to the mean wind at midspan whilst inclining away from the toll torque of any shift of ther upper wind[2]. (Instead a sweptback wing doesn't cutout but basically diverges in high winds).

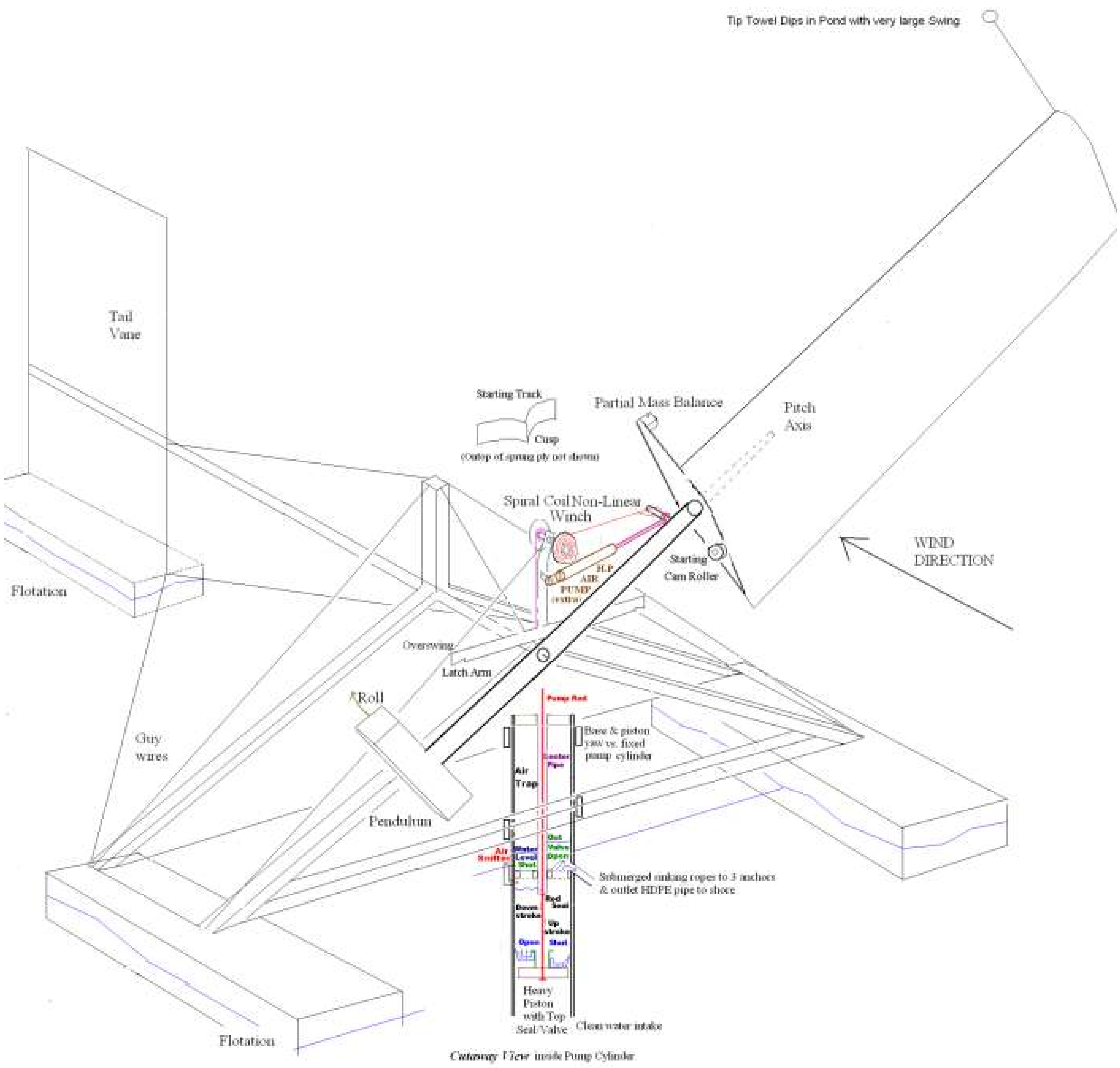


Fig 1 Perspective of Floating *Flo*'Pump with pump down and up stroke crossections (cFlopump.tif)

The rotary multiblade windpump loses all the kinetic energy of the swirl reaction to its high torque [3], which still can't turn over its crank to start in the best wind for its stroke. For a typical wind regime it can only usefully pump just <u>10%</u> of the work of a Betz ideal windmill [4]. Duncan's fixed cycle wing engine would also stall against the fixed head of a piston pump. The free amplitude of flutter instability is inherently suited to reciprocating a pump whose stroke needs to vary to efficiently capture the changeable wind[4]. The Flutter rectangular wing free to pitch (360°) on top of a roll pendulum winds a spiral winch pump (Figs 1 &2) with stroke varying as almost the cube of roll amplitude. So the pump stroke increases strongly with roll, not loading a starting swing in only 1m/s, but still absorbing up to ½ of the Betz wind energy swept in a big roll in a good wind. Compressing an aircylinder on the return stroke is very simple but not so supra-linear so a flow rate of about 3 cfm to a pressure of 4 atm for 250W was the max for a total 400 peak pumping W (eg 4L/s @10m head) in 3.5m/s by an 5.5 x1.37x.08 m 20 kg wing. Wind gradient and the non-linearities of the stiffening of the single-acting non-linear pumping, large amplitude pitch, and crosswind roll speed exceeding windspeed all increase the power[5] and the cutout causing a hysteresis vs. the linear restart wind. Key to curbing any overswing is that flutter pitch and roll are partly in phase so the Flo'Pump's tip balance rod makes first contact with the pond to wet its absorbent material and temporarily reduce the tailheaviness that flips the wing. A simple cusp track ensures self-starting from small amplitude in very light and steady winds but misses wing cam contact at bigger amplitude or when excess winds blow it back. The *Flo*'Pump prototype Fig 1 floating around its pump cylinder with upper air chamber pumped through its underwater outlet pipe to shore and overland powerfully and reliably for 10 years with its catamaran righting moment limiting the stresses [8] . It needs help with commercialisation or adoption in the Third World or making very large prototypes to assist pumped hydro.

The existing big powerful Flo'pump was designed to be moved along lakeshores and river valleys to pump more than 7m high, where a fanmill confined to one location would have trouble sucking and need a stuffing box. For shallow flat shores, but with stable water level , another simpler Flo' pump seems possible, dispensing with the pump rod and seal, the airchamber and the anchors. Instead a standpipe rises up to the winch to smooth the output to shore to allow primary irrigation of low fields with perhaps a ram there feeding smaller higher further fields. The lower part is a rigid steel casing/cylinder internally excavated as it is driven into the bottom. It can have an angled-down side pipe to shore just above the lowest waterlevel so the Flo'pump can still float around it. Onto its top above the windmill yaw ring is slipped a PVC sewer standpipe. Its internal rubber ring joint adjusts to the pitch, roll, heave and yaw of the winch. The side pipe could instead exit here onto the base to stroke the full length of the pile and to yaw say 120 yaw eg +/-60 from wind parallel to shore. From a check valve above the sediment, the inlet pipe descends welded alongside the pile to the bottom side for the lowest stroke. A universal floating base will be designed for this and the pressure pump option.

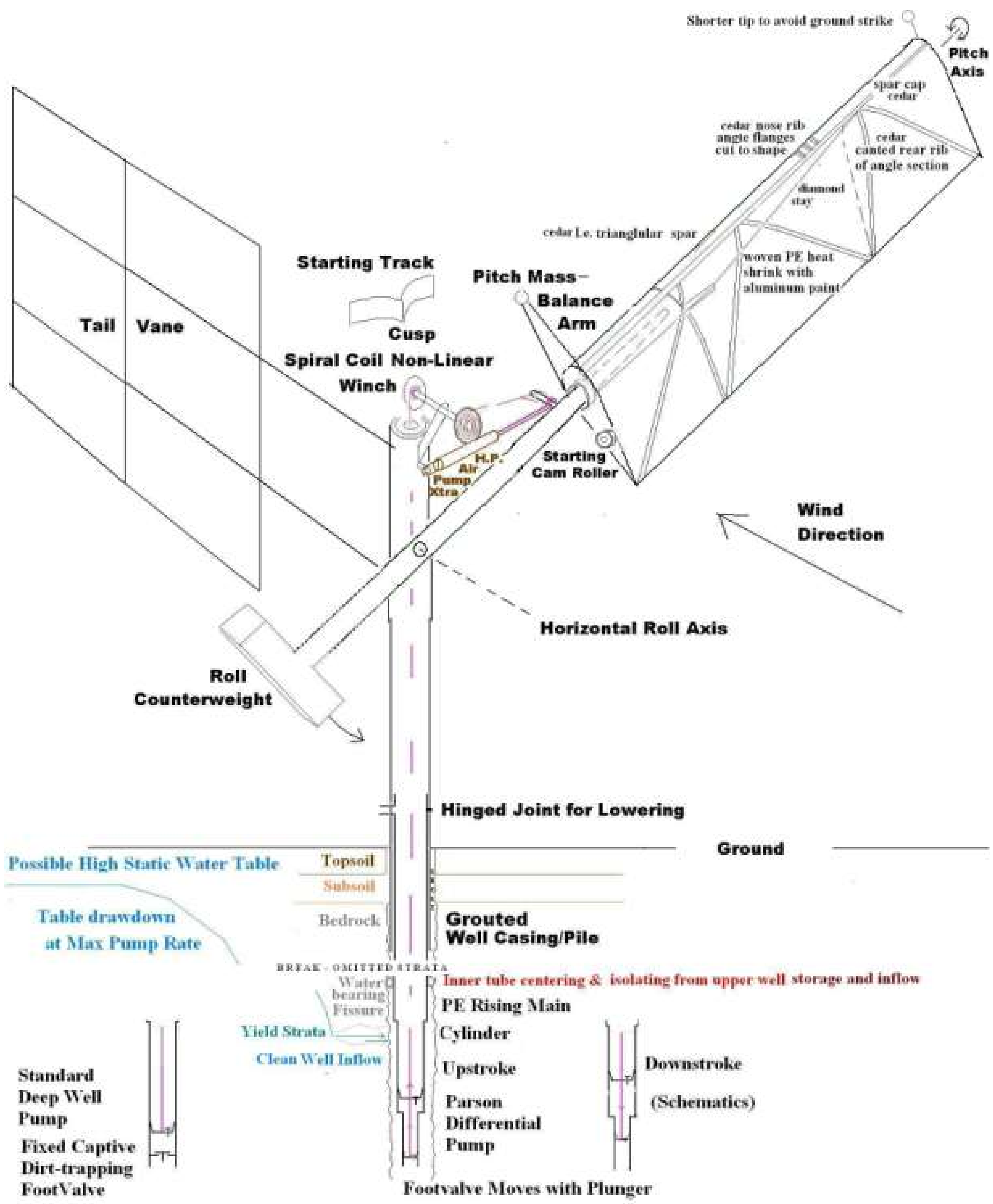
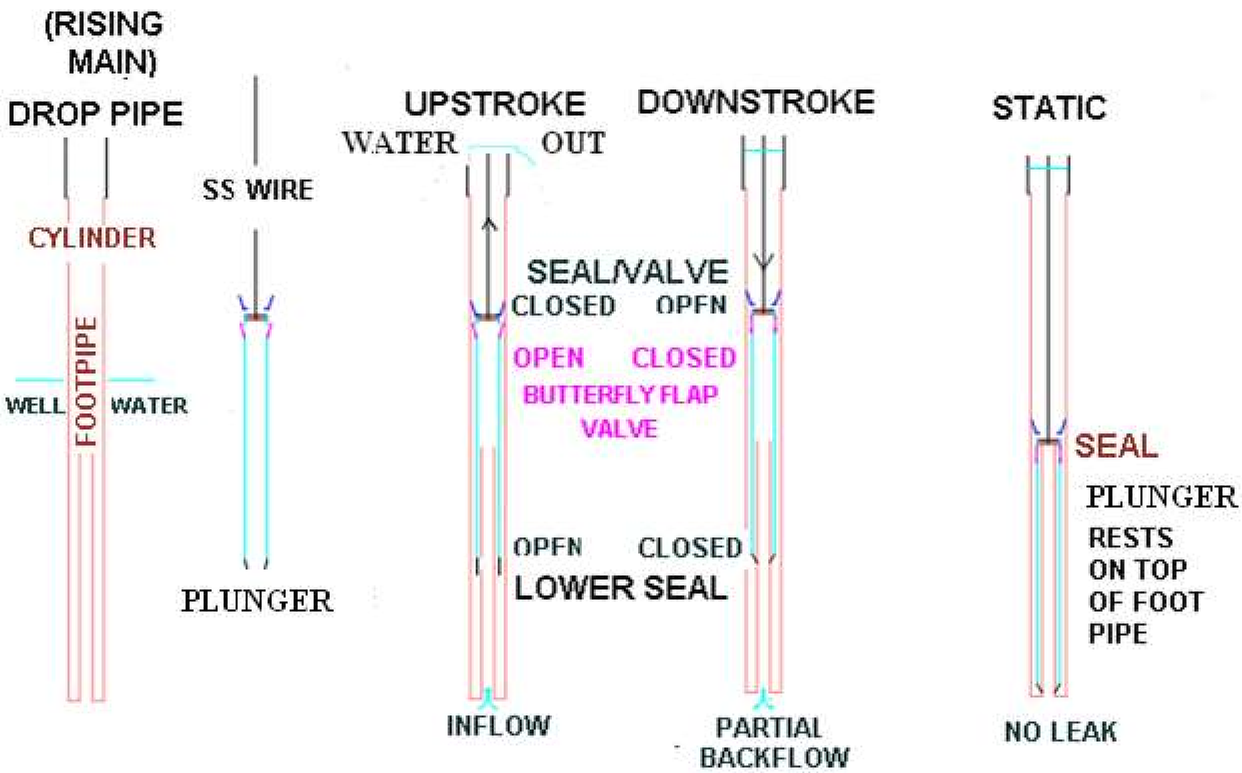


Fig 2 Schematic of the Flutterwell Pump, the Well-mounted FlutterWing Pump cFar4.jpg

The Flutter*well* base in Fig 2 uses the steel well casing as a short foundation pile. A new model [9] has shown prolonging the nose down pitch in a roll overswing disrupts the wingflip so the next opposite roll is safely smaller. Then after a long calm the Flutterwell could pump enough easy large, but safe, strokes at lowhead to drawdown the well and a high static watertable to a sustainable yield. But such well storage and near-well groundwater drawdown transients can be largely eliminated by a centering sanitary seal inflated between the rising main and the well just above the strata yielding water. Some of the wind's varying power is inevitably wasted in the varying head of groundwater resistance.

The novel differential area pump keeps the pumpwire in tension for downward accelerations up to its upper/lower area ratio>1 times Newton's g. Pulling its plunger with all the seals and valves out of the cylinder releases the water column for easy retrieval and maintenance. Its design for low internal flow constriction Fig 3 uses Fig 1's single part seal valve this time just above a perimeter rubber flap valve over a grid of holes, and a ring seal inside the bottom of the plunger. A snifter, surface valve and airtank have been tested at the wellhead for overland pumping overcoming air absorption and indeed outputting some compressed air. A spinoff is a powerful well footpump[10], but a fluttering watermill is elusive, even with the dominance of static gravity imbalance in pitch and roll flutter in water.

Fig 3 Detail of the actual differential well windmill and footpump



. Despite being ubiquitous in early aircraft, flutter has scarcely been a problem in marine hydrofoils. The ratio of foil mass to the virtual mass $\boldsymbol{m}$ of the circumscribing fluid cylinder is much lower with the 700x water density. Despite the aircraft premium on light weight, buckling limits due to large spans keep wing mass ratios well above one even for typical wings (as the size of airships would also indicate). Cross-over flutter calculations of typical hydrofoils by eminent aeroelasticians [11] reinforced their empirical lower flutter limit of roughly equal real and added mass. Such voluminous linear computational & experimental flutter literature has made developing binary flutter as an oscillating windmill or watermill very difficult as well as cross- disciplinary. An upside down version of Fig 1 with two opposed ferrocement blades could be not be adjusted to flutter when towed through water.

So the basic heaving fluttermill of chord $\boldsymbol{c}$, virtual mass/unit length $\boldsymbol{m}$, that is free to pitch γ about an axis $\boldsymbol{ec}$ ahead of the ¼ chord aerodynamic center has been analysed in a series of papers [2,12,13]. The lack of γ mechanical stiffness allowed a first ever algebraic solution of binary flutter drawing all the neutrally stable frequency ω contours in total mass imbalance $\boldsymbol{xmc}$ vs pitch inertia $\boldsymbol{jmc}^2$ space. All contours pass through a nexus $\mathbf{N}=(\boldsymbol{mo}^2,\boldsymbol{mo})$ of the same total inertia and imbalance as just $\boldsymbol{m}$ mislocated (too far aft) at the ¾ chord point $\boldsymbol{o}$ behind the pitch axis [12] where the nominal upwash $\boldsymbol{U}_{3/4}$ and so the wake and circulation vanish for all ω. Let $\mathcal{m}$ be the apparent mass/span of the finite (thin) wing $\boldsymbol{m}= l\mathcal{m},\ l\approx 1+\frac{1}{2}A^{-2}$. Then *a priori* for a 2D ¾ chord line offset $\boldsymbol{o}$ to the pitch axis the flow is potential at $\mathbf{N}=(\mathcal{m}o^2,\mathcal{m}o)$ because again $\boldsymbol{U}_{3/4}=0$ always all along such a finite wing.

Actually virtual and real (structural) $\boldsymbol{j/x}$ are closer to the pitch axis and so are on the left side of the 'ray' from (0,0) to $\mathbf{N}$. Even an understressed solid steel hydrofoil is short of the low frequency limit line where flow stiffness dominates inertia. With pitch axis lead $\boldsymbol{e}>0$, this steady lift "lfl" line through $\mathbf{N}$ extends downwards in $\boldsymbol{x}$ with $\boldsymbol{j}$ to reduce the tailheaviness $\boldsymbol{x}$ for flutter (even to negative $\boldsymbol{x}$ or noseheaviness.) Generally the larger flutter zone of higher $\boldsymbol{m}$, $\boldsymbol{j}$ and $\boldsymbol{j/x}$ (partially mass-balanced wings) to the right of the nexal ray and above the lfl allows more heave/pitch for heavier-than-air Wing'd Pump power with high wind $\boldsymbol{V}$ cutout in pitch and roll.

The binary discriminant σ of this biquadratic in $\boldsymbol{j}$ and $\boldsymbol{x}$ is proportional to the discriminant δ of the quadratic in $\boldsymbol{e}$ for pure pitch flutter[13] :

$$4F^2\sigma/k^4=\delta=(2g-F-\tfrac{1}{2})^2-2F=\{2g-F-\tfrac{1}{2}-\sqrt{2F}\}\,[2g-F-\tfrac{1}{2}+\sqrt{2F}]=\{2g-(\sqrt{F}+1/\sqrt{2})^2\}[2g-(\sqrt{F}-1/\sqrt{2})^2] \qquad (1)$$

where $\boldsymbol{T}=\boldsymbol{F}-i\boldsymbol{G}=\boldsymbol{F}-i\boldsymbol{kg}$ Theodorersen's celebrated 1933 1D wake function of reduced frequency $\boldsymbol{k}=\omega\boldsymbol{c}/\boldsymbol{V}$. Now as $\boldsymbol{k}\uparrow\infty$, $\boldsymbol{F}\downarrow\frac{1}{2}+\frac{1}{4}/\boldsymbol{k}^2$ and $\boldsymbol{G}\downarrow\frac{1}{4}/\boldsymbol{k}$ so by extraordinary cancellation $\sqrt{2g}>\sqrt{F}-1/\sqrt{2}$ making the 1D high frequency contours ellipses, (asymptoting linearly to a 2.5$\mathbf{N}$ "sweet spot" on virtual ¾ chord pitch ray from the nexus $\mathbf{N}$ to 4$\mathbf{N}$[13]). Any 2D change of this 1D lower limit of $\boldsymbol{F}$ from ½ will open the high frequency contours into hyperbolae.

As $\boldsymbol{k}\downarrow 0$ a negative Ln singularity in the 1D $\boldsymbol{g}$ and so in the net pitch damping allows pure pitch flutter at $\boldsymbol{k}_z\leq .087$ of {}≤0 for δ<0, (with very high pitch inertia and further fluid power loss fed into the wake vortices by the drag work.) This hyperbolically repels the $\boldsymbol{k}\approx\boldsymbol{k}_z$ binary contours, even bizarrely back to very large negative imbalance but no inertia at about $\frac{1}{2}\boldsymbol{k}_z$ [13], Here 2D finite aspect ratio A is added for a vital reality check to [13].

…………

In all, this paper has given the state of the art of power flutter, from the proven Flo'Pump to the Flutterwell pump which needs testing on real wells to final proof of the impracticality of pure pitch flutter and finite aspect ratio extension of the search for hydrofoil variants.

References

1. Duncan, J.W. The fundamentals of flutter (1948) , Ae.Res.Co. R&M No. *2417*, November, 1-36
2. Farthing, SP. Binary Flutter as an Oscillating Windmill :Scaling and Linear Analysis *Wind Engineering* 2013 37 (5) : 483-99
3. Dixon, JC. Load Matching Effects on Wind Energy Convertor Performance p 418-421 in *Future Energy Concepts* , IEE Conference Publication London 1979
4. Farthing, SP. Visualisation and analysis of Torque Reacting Flow through Starting FanMill *Wind Engineering* 2010Vol 35 (5).: 625-634
5. Farthing, SP. The Flutterwing Pumps: Design, NonLinearities, & Measurements *Wind Engineering* 2014 38 (2): 217-31
6. Farthing, SP. FWP Leading edge smoke youtube.com/watch?v=16kB6p-kcC0 1993
7. Platzer, MF & Young, J. Numerical Simulation of Fully Passive Flapping Foil Power Generation *AIAAJ* 2013 51(11) : 2727-39
8. Farthing, SP. Wing'd Pumps : CC scripted video youtube.com/watch?v=6zIj7LCtX0U (2012)
9. Farthing, SP. http://econologica.org/yardpump.html
10. Farthing, SP. Flutterwell overswing prevention youtube.com/watch?v=A9HR6Ml6iWk (2020)
11. Ashley, HA. & Dugundji, J. Aeroelastic Stability of Lift Surfaces in High-Density Fluids *J. Ship Research* 1959 3(1) : 10-28
12. Farthing, SP. Binary Flutter Solution for Fluid Power *Journal of Aerospace Engineering*, (ASCE) 2018 31(3): 1-11
13. Farthing, SP. Exact Pitch and Heave Flutter for Complex Theodorsen function *Aero. Journal* 2019 123(1265), p.1053-1074
14. Jones, RT. The Unsteady Lift of a Finite Wing 1939 NACA-TN-682
15. Boutet, J & Dimitriadis, G. Unsteady Lifting Line Theory Using the Wagner Function for the Aerodynamic and Aeroelastic Modeling of 3D Wings Aerospace, 2018 - orbi.uliege.be
16. Andrianne, T. Unsteady Theodorsen Aerodynamics http://www.ltas-aea.ulg.ac.be/cms/uploads/AERO0032-1_2015-16_Lecture3.pdf
17. Schlichting, Hermann. & Truckenbrodt, Erich. (1979) Fig 3-43 *Aerodynamics of the airplane*. New York : McGraw-Hill
18. Wolfram Research, Inc., Mathematica, Version 7.0, Champaign, IL (2008).
19. Abramowitz, M.; Stegun, I. A., eds. (1972), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, New York: Dover Press, ISBN 978-0-486-61272-0 p 378 (130)and p 375 (127)
20. Jumper, EJ &. Hugo, RJ. Finite wings undergoing rapid unsteady motions -Theory 2012 Journal of Aircraft 31, (3) p495-502
21. Hoerner, SF & Borst H. 1975. *Fluid-dynamic lift: practical information on aerodynamic and hydrodynamic lift* Bricktown N.J.
22. Farthing, SP. Binary Flutter in Water youtube.com/watch?v=NDm78DOcEOM

Symbol Table

subscripts $_\pm$ evaluated at $y/s = t \pm r$

$a = 2\pi f$ 1D lift slope, static a_0

b semichord c chord of foil, c_0 at root of ref elliptic planform

ec trail of quarter chord behind pitch axis in chord

f correction factor for Re

$g = G/k$ negative pitch damping rate of G

h heave of pitch axis

$h_{3/4}$ heave of the ¾ chord point

i square root of -1

j pitch inertia/mc^2, $\jmath$ pitch inertia/$\mathcal{m}c^2$

k reduced frequency based on chord $\omega c/V$

m added mass of foil/unit length $= l\mathcal{m}$

$\mathcal{m}$ added mass of wing /unit length

$n = e + ¼$ chords from pitch axis to midchord.

$o = qc$ distance from pitch axis to ¾ chord $q = e + ½$

$p = a/4A$ parameter in I_t

r fraction of semispan where trailing induction is average

s wing semi-span

t center of trailing vorticity as fraction of semispan

$\mathcal{t}$ time

x pitch imbalance/ mc

$\mathcal{x} = lx$ pitch imbalance/ $\mathcal{m}c$

w distance from the ¾ chord point

$y = e+¼/F$ chordal distance parameter

A or AR Aspect ratio $2s/c$

$B(y)$ steady lift spanwise function

B_n steady lift Fourier coefficient of sin $n\theta$

C 1D Correction factor for I_s

D induction by wake vortex strand of unit strength $_{\mathrm{I(nternal)},\ \mathrm{E(xternal)}}$

$E_1(u)$ =Exponential Integral function $E_1(u) = {}^{\infty}\!\int_1 \mathrm{d}t\ \mathrm{e}^{-ut}/t$

I back-induced factor of nominal upwash (on average)

I_t from trailing vorticity , evaluated at ¾ chord, steady I_q

I_s shed vorticity, evauated at midchord

I_n Back induction of nth Fourier spanwise mode of steady lifting line

$M(S)$ A^{-2} unsteadiness factor in induction of external trailing vortex

N =1+ I_q is the steady lift divisor due to A N=1+ 2.13fA^{-1} +1.11f^2A^{-2}

N Nexus point $(j,x)=(q,q^2)$

$R(S)$ primary unsteadiness factor in induction of external trailing vortex

$S = \omega\Delta y/V$ or the reduced frequency based on spanwise separation Δy,

$T = F-iG = F-ikg$ complex Theodorsen $T \approx f/(1+I_t+C(v)I_s)$ $T_{\mathrm{J(ones)}}$. $\bar{T}=Tl$

U upwash $_{¾}$ evaluated at ¾ chord point or $_e$ at pitch axis.

V flowspeed

ρ Fluid density

γ pitch angle

φ apparent ¾ chord angle of attack

ϕ net ¾ chord angle of attack

Γ complex amplitude of pitch

φ Nominal ¾ chord Angle of attack ignoring wake induction

σ Discriminant of the neutral stability quadratic eqn in j,x $\delta=4F^2\sigma/k^4$

ω circular frequency in radians of phase/unit time